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Discrete time reaching law based sliding mode control: a survey

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***Abstract*—The reaching law approach comprises of first specifying the required evolution of the sliding variable. Then, a sliding mode controller that enforces this evolution is designed. The main advantage of this method with respect to "classical" sliding mode control is better control of the plant dynamics and state constraints during the reaching phase. In this paper, a review of a number of new works on reaching laws for discrete time systems is presented. The differences and similarities between them are discussed.**

Keywords—reaching law, discrete sliding mode control

I. Introduction

Sliding mode control is a well-developed method that is suitable for a large spectrum of non-linear, time-varying, uncertain systems. Its main advantages are the robustness with respect to external disturbances and efficient implementation. Sliding mode control was at first proposed for continuous time plants. Nonetheless, as in modern days the controllers are almost always implemented in digital hardware (microcontroller, DSP, PLC, etc.), sliding mode control for discrete time systems quickly became a crucial topic of research. The main concept in sliding mode control is to choose a hypersurface in the state space to which the motion of the state (representative point) will be confined. The shape and orientation of this hypersurface will determine the dynamic behavior of the obtained closed loop system. A switching variable is a notion closely related to the chosen hyperplane. This variable is equal to zero if the representative point is on the hypersurface, is positive for states on one side of the hypersurface and negative for states on the other side. The absolute value of this variable increases as the state is farther away from the hyperplane.

A controller is derived that, relying on the value of the sliding variable i) in finite time will make the representative point reach the hypersurface, in what is known as a reaching phase, ii) maintain the representative point on the hypersurface (or in its immediate neighborhood) during the so-called sliding phase. This problem can be solved using one of two basic approaches. The primary one starts by putting forward a controller, and then demonstrating that the above requirements are satisfied if it is applied. The latter manner relies on the reaching law, and is the focus of this survey. Using this technique the arrangement of operations is in some sense inversed. It begins with specifying the reaching law, namely the required convergence of the sliding variable to zero. For continuous time plants this is typically a relation between the time derivative of the switching variable and its present value. For discrete time plants, the reaching law generally describes the desired value of the switching variable in the next time instant based on the present value of this variable. Then, a controller is derived, that will ensure that the sliding variable evolves in accordance with the reaching law.

The reaching law methodology has two main advantages. Firstly, after designing the controller there is no need to prove that sliding mode is ensured, as this follows directly from the reaching law. Secondly, the convergence of the state to the switching hypersurface is explicitly taken into account, which allows for better control of system dynamics and state constraints during the reaching phase.

II. Switching Reaching Laws

The reaching law technique was first postulated in [15] for continuous time plants. Three reaching laws: power rate, fixed rate and fixed plus proportional rate were proposed and compared. Then the authors in [16] extended their results to the discrete-time case. As in discrete systems, the ideal sliding motion cannot be enforced, they proposed a definition of a quasi-sliding motion, which is expressed in the following three requirements:

1) The representative point must progress monotonically toward the sliding hypersurface from any initial position and in finite time cross it.

2) Following the first crossing of the switching hypersurface, the state of the plant must pass through it subsequently, in every following discrete step.

3) The amplitude of the oscillations resulting from this „zig-zag" motion cannot increase, and the representative point must remain in a known neighborhood of the sliding hypersurface.

Next, the authors postulated a reaching law in which the convergence rate is governed by two terms: one constant, and one proportional to the present value of the switching variable (analogous to the "fixed plus proportional" reaching law developed in [15]). The postulated evolution of the switching variable for the perturbed plant is

$$s\left[(k+1)T\right]=(1-qT)s(kT)+ \\ -(\varepsilon+S_2+F_2)\operatorname{sgn}\left[s(kT)\right]+\tilde{S}+\tilde{F}-S_1-F_1, \quad (1)$$

where $q \in (0, 1/T)$ and $\varepsilon > 0$ are design parameters. The terms $\tilde{S}$, $\tilde{F}$ are the unknown effects of the modelling uncertainties and external disturbances on the s variable respectively. The terms S_1, F_1 represent the mean values of $\tilde{S}$, $\tilde{F}$ and S_2, F_2 correspond to the maximum deviation of those terms from their means.

It has been demonstrated in [2], that the parameters of (1) must satisfy the condition

$$qT\varepsilon T\big/\left[2(1-qT)\right]>S_2+F_2 \quad (2)$$

in order to assure the existence of quasi-sliding motion.

The reaching law (1) is frequently utilized in the current works both in its original [19], [25], [26], [32], [38] as well as modified [17], [22], [29], [33], [34], [36], [40] form.

In [38] the influence of time delays on discrete time systems was analyzed. A discrete time state predictor was proposed. Then it was combined with a sliding mode controller based on (1). The results were demonstrated by computer simulations.

On the other hand, in the work [25], the control of a so-called smart structure was considered. Such a structure consists of a host structure on which sensors, actuators and controllers are placed. The main aim of control is to limit the oscillations whose frequencies are close to one of the resonant frequencies of the construction. In order to achieve this goal, a modified quadratic control quality criterion was proposed. The oscillations that are close to the resonant frequencies have a larger impact on this criterion, than other oscillations. In order to create an optimal controller minimizing this criterion, the state space representation of the plant was extended, incorporating the outputs of bandpass filters. Using this extended representation, a "classical" LQR criterion is applied, with increased weights corresponding to the outputs of the filters. Moreover, as in the considered systems, the states (deformations of the structure elements) are difficult to measure, a multirate output feedback technique, that estimates the states based only on output information, was used. The last step was the design of the sliding mode controller, which was performed using the reaching law proposed in [16]. Experimental tests on a laboratory stand, as well as computer simulations were performed, to support the theoretical results.

In the paper [19], the reaching law postulated in [16] was also combined with the multi-rate output feedback, to remove the necessity of measuring every state variable. As in this approach, the exact state is replaced by its estimate, it leads to an increase of the width of the quasi sliding mode band. The authors calculate the new width. The operation of the algorithm is verified in computer simulations. Conversely, in [32] the reaching law put forward in [16] is applied to obtain a predictive sliding mode controller.

In the work [26], the impact of the discretization period on sliding mode control of a MIMO system is considered. The authors first considered the scenario, in which every element of the feedback gain matrix changes between two values, in accordance with the sign of the corresponding switching variable. These values must not be too small as they have to counteract the external disturbances influencing the system, or too large (which is not true in control of continuous time plants), or they would result in crossing the sliding hyperplane and inducing oscillations in the system. The changes of both the upper and lower bound, with respect to the discretization time were calculated. As the discretization period tends to zero, the behavior of the plant gets closer and closer to the continuous one, and one of the bounds tends to $\pm\infty$. As the discretization period gets larger, this bound gets closer to the other one, which is mainly a function of the disturbances impacting the system. At a certain value of the discretization period, both bounds achieve the same value, thus it is impossible to assure stable sliding motion for larger discretization periods. The authors propose a control signal, which is a sum of linear feedbacks and a nonlinear function used to ensure convergence towards the sliding hyperplane, in spite of the disturbances influencing the system. Next, the authors propose two controllers, one based on the reaching law from [16] and the other on modifying the reaching law by introducing the saturation function in place of the sign function, to minimize induced oscillations. Finally, results of computer simulations of all the three controllers are presented.

On the other hand, in [17] a reaching law was postulated, in which the speed of convergence of the switching variable is a certain function of its value. The function is proportional to s if the absolute value of s is small, which reduces chattering. The effect of disturbances on the plant controlled in this fashion is investigated, and the size of the neighborhood of the switching hypersurface, to which the state will converge is derived. As an illustrative case, the authors propose using a function that corresponds to the reaching law from [16] if the distance from the switching hypersurface is large, and a proportional function in close vicinity of the switching hyperplane. The results are verified by position control of a direct current motor, in which the impact of the discretization period on the control accuracy was also checked.

The approach proposed in [17] was then built upon in [29], by introducing an additional integrator of the sliding variable. This allowed the DC motor servomechanism to follow a parabolic position trajectory. In order to prevent the integrator from saturating (which is known as windup), its action is activated only in close vicinity of the sliding hyperplane. Moreover, the authors compensate the disturbance using a feedforward action, which allows to follow a reference trajectory that depends on the third power of time. The authors calculate the steady state error analytically and compare it to the one obtained in [17].

In the paper [22] a sliding mode controller is used to stabilize the voltage in a DC power network. The authors first consider using the approach from [16], but this would result in excessive chattering. Therefore, they propose a modification of the reaching law in close vicinity of the sliding hyperplane. When the value of $|s|$ drops below a certain threshold, the constant value ε in (1) is replaced by the term $\beta|s(kT)|^{\lambda}$, where β and λ are constant parameters. Because this term decreases as the sliding variable tends to zero, this modification results in a significant reduction of chattering. It is then demonstrated, that in case of no external disturbances, the quasi sliding motion defined in [16] is enforced. Moreover, as most of the reaching phase takes place outside of the sliding hyperplane vicinity, this modification does not noticeably increase the duration of the convergence to the sliding hyperplane. The action of the proposed controller was verified on a laboratory stand.

On the other hand, the authors of [34] proposed the following reaching law

$$s\left[(k+1)T\right]-s(kT)=-qTs(kT)-\left|s(kT)\right|T\operatorname{sgn}\left[s(kT)\right]/\rho \;. \quad (3)$$

The constant parameter ε in (1) is replaced by the term $|s(kT)|/p$. However, the equation (3) is overcomplicated, as its right hand side could be rewritten as $-(q + 1/p)Ts(kT)$. This means, that there is no need of selecting q and p individually, as only the value $(q + 1/p)$ will influence the controller action. Therefore, one can consider (3) as a modification of (1) in which $\varepsilon = 0$ and $qT > 1$ is allowed. The authors compare the reaching laws (1) and (3) in computer simulations.

In the paper [36] the control of a PMSM speed is considered. It is assumed, that the position of the rotor is obtained from an encoder, and the angular velocity is estimated using the Euler method. A reaching law is proposed, that divides the state space into two regions. In the first one, which comprises of points that are farther from the hyperplane than a certain threshold value, the desired convergence rate of s is large. In the region closer to the hyperplane the desired convergence rate is reduced. This approach corresponds to altering (1), by setting $q = 0$ and altering ε between two values, determined by the magnitude of the switching variable. Using this approach, one can obtain short convergence time, as ε is large in the "outer" region, while still keeping chattering to an acceptable level, since ε is reduced in the vicinity of the hyperplane. These theoretical finds are verified experimentally on a test stand.

In the work [40] the authors formulated the subsequent reaching law that is based on two power functions of the switching variable:

$$\begin{aligned} &s\left[(k-1)T\right]-s(kT)=-qTs(kT)+\\ &-\varepsilon_1 T\left|s(kT)\right|^{\alpha}\operatorname{sgn}\left[s(kT)\right]-\varepsilon_2 T\left|s(kT)\right|^{\beta}\operatorname{sgn}\left[s(kT)\right], \end{aligned} \quad (4)$$

where ε_1, ε_2 are positive parameters, $q \in (0, 1/T)$, $\alpha \in (0, 1)$ and $\beta > 1$. One can observe, that equation (4) smoothly transitions between two different power reaching laws. When $|s(kT)| < 1$ the term with parameter α dominates, conversely if $|s(kT)| > 1$, then the term with parameter β obtains larger values. Moreover, parameter q, which was constant in all the above-mentioned works can take different values for $|s(kT)| > 1$ and $|s(kT)| < 1$. The reaching law (4) can be viewed as an extension of the one proposed in [22]. The difference is that the transition between two convergence rates is more smooth. Moreover in the work [22], in one of the regions, the power function was replaced by a constant term. The authors demonstrated, that using (4) guarantees the existence of quasi sliding motion if there are no disturbances acting on the plant. Then, the two parts of the reaching phase are analyzed in more detail, the first one from the initial state to the neighborhood of $s(kT) = 0$ and the second one, from this vicinity to $s(kT) = 0$. In order to employ the equation (4) to real systems the authors added a neural network, in order to approximate the values of the disturbance, which in turn allows to compensate it. Extensive computer simulations were performed, in which four reaching laws were compared: law (1), single power reaching law (namely (4) setting $\varepsilon_2 = 0$) and the reaching law (4) both in its base form as well as extended by disturbance estimation.

A similar approach was used in [13], where the reaching law (1) is altered by exchanging the constant parameter ε with a function of the root of the absolute value of the switching variable, namely using (4) with $\varepsilon_1 \in (0, 1)$ and $\varepsilon_2 = 0$. In computer simulations, the authors compare their approach with (1) and confirm that a significant reduction in quasi sliding mode band width is achieved. In [39] the reaching law used in [13] is applied in order to realize maximum power point tracking of a wind turbine. The sliding mode controller determines the duty ratio of the boost converter, which in turn affects the power drawn from the turbine. The designed controller is examined in contrast with a sliding mode controller without a reaching law, and the perturb and observe method in laboratory experiments. The wind turbine is "simulated" by an external induction motor, under control of its own microcontroller. The results show, that although all controllers converge to the maximum power point, the proposed approach offers the smallest voltage ripple and the shortest settling time.

In the paper [33] the problem of disturbance estimation was considered. The reaching law (1) was extended by disturbance compensation, assuming that the disturbance rate of change is small. The authors have moreover omitted the necessity of the state crossing the switching hypersurface in each control step. Instead of this, they calculated the upper bound of time that separates two consecutive crossings of the hyperplane. Both of the modifications allowed to obtain a smaller size of the quasi-sliding band than in [16], which plainly corresponds to better robustness of the plant with respect to perturbances. These claims have been verified in computer simulations.

On the other hand, in [30], a sliding mode controller was used to control the speed of an asynchronous motor. To limit the maximum angular speed, a nonlinear hypersurface was selected. Next a control signal was calculated, that guarantees asymptotic convergence to the vicinity of this hypersurface. Computer simulations as well as experiments illustrate the advantages of the presented solution over the ones shown in [16] and [35]. The advantages lie mainly in reducing the oscillations of the torque generated by the motor. Nonetheless, the obtained results are not general, but are limited to the considered system.

In [6] the authors proposed the subsequent reaching law

$$s\left[(k+1)T\right]=\left\{1-e^{-\left[s(kT)/s_0\right]^{2m}}\right\}\left\{s(kT)-s_0\operatorname{sgn}\left[s(kT)\right]\right\} \quad (5)$$

The first expression on the right hand side of (5) decreases when the sliding variable value gets close to zero. This reduces the action of the switching term, and limits the undesirable chattering. Moreover, reaching law (5), in contrast to many previously presented ones, ensures an upper limit on the switching variable rate of convergence. For many systems this corresponds to limiting the magnitude of the controller action. The reaching law (5) is also modified to be applicable to systems with parameter uncertainties and disturbances. The existence of the quasi sliding motion, and the uniform ultimate boundedness (assuming that s is chosen to obtain dead-beat dynamics in sliding mode) of the system are proven analytically. Finally, the authors compare their solution to the reaching law proposed in [16] in computer simulations of the nominal system, as well as the perturbed one. In both scenarios, the new approach results in a narrower width of the quasi sliding band as well as smaller values of IAE and ISE control quality criteria. Then in [7], the authors use a slightly modified form of reaching law (5), to control the flow of goods in a supply chain. The important properties, such as full consumer demand satisfaction are demonstrated in Matlab simulations as well as proven analytically.

A relatively new and interesting research direction is the combination of the reaching law approach with sliding variables of higher relative order. In a "traditional" discrete time algorithm, the control signal affects the sliding variable after a single discretization period. Similarly, a r-th relative degree variable is influenced by the control signal only after r control steps. In this way, the values of $s(k+1)$, $s(k+2)$, …, $s(k+r-1)$ are already determined at instant k and can be used to generate the control signal $u(k)$. The choice of a higher order sliding variable can be either dictated by a lack of measurement of state variables that are directly influenced by the control signal, or can be a conscious choice of the control designer.

In [5], the authors first proposed modifying the reaching law (1) by introducing a function $h(s)$ in place of the term $(1 - qT)$, where $h(s) = 1$ for $|s| \geq s_0$ and $h(s) = |s|/s_0$ for $|s| < s_0$, where s_0 is a positive design parameter. This modification allows to limit the rate of convergence of the sliding variable, and thus also the control signal, for big initial values of s. The authors demonstrated, that using their approach ensures, that the quasi-sliding motion as specified in [16] exists. The region around the desired value, to which the output of the system will converge was also analyzed and derived. The reaching law was adopted to the case of second order switching variable s_2, in such a way, that the favorable properties mentioned earlier still hold. The advantages of the second order sliding variable control, namely smaller quasi-sliding mode band and smaller output error, were demonstrated in computer simulations.

In [4] the reaching law (1) was generalized for arbitrary degree sliding variables. The choice of sliding variable of degree r was facilitated by a transformation of the system to the Frobenius form. Moreover a time-varying sliding hyperplane has been used to eliminate the reaching phase. Then it was demonstrated, that the increase of sliding variable relative degree results in better control precision, namely smaller errors of state variables and narrower width of the quasi-sliding band.

III. NON-SWITCHING REACHING LAWS

In the work [3], the description of a discrete time quasi sliding motion proposed in [16] was modified, by removing the requirement of crossing the hypersurface in every successive discretization step. It has been assumed, that the external disturbance and the effect of parameter uncertainties satisfy the matching conditions. The following reaching law was proposed

$$s\left[(k+1)T\right] = d(kT) - d_0 + s_d\left[(k+1)T\right], \tag{6}$$

where $d(kT)$ is the effect of external disturbances and modelling uncertainties on the switching variable, d_0 is the mean value of $d(kT)$ and function $s_d(kT)$ must satisfy the following three conditions:

1) The initial value $s_d(0) = s(0)$.

2) The function $s_d(kT)$ never changes its sign.

3) The value of $|s_d(kT)|$ for $|s_d(kT)| > 2\delta_d$ decreases at each step, at least by $2\delta_d$, where δ_d is the greatest possible difference between $d(kT)$ and d_0.

According to the third requirement, at some finite k^* the inequality $|s_d[(k^* - 1)T]| \leq 2\delta_d$ will be satisfied and then $s_d(kT \geq k^*T) = 0$. In a presented example $s_d(kT)$ was chosen as a linear function decreasing to zero. It has been demonstrated, that such a strategy ensures better robustness with respect to disturbances, than (1). Moreover, a disturbance compensation term was added to the proposed strategy, which is built on the assumption of bounded change rate of disturbance. It has been verified, that this allows to further improve the robustness.

In [12] the reaching law proposed in [3] was extended to switching variables with relative degree two. Moreover, a modification of equation (1) for relative degree two switching variables was presented. It has been demonstrated, that this method can enhance the robustness of the plant in both cases.

The work [18] also builds upon the results of [3]. The original method required measurement of the whole state vector, which can be difficult for some plants. Because of this, the authors of [18] combined it with the multirate output feedback technique. This replaces the state vector with its estimate, and slightly increases the quasi sliding band. The new size of the band was calculated, and the strategies were compared using computer simulations. The paper [24] presents results that are similar to [25] which were covered earlier. The difference is that instead of (1), the reaching law (6) was used. The authors demonstrated that this change allows for better damping of the oscillations.

Another modification of the reaching law (6) was proposed in the work [37], in which the value of $s_d(kT)$ is related to the current value of $s(kT)$. This allowed to further shorten the duration of the reaching phase. However, it must be said, that the disadvantage of this approach is a reduction of robustness during the reaching phase.

In the work [8] both switching and non-switching reaching laws were analyzed. Firstly, a modification of reaching law (1) was proposed. The fixed convergence rate q was replaced by a function of the switching variable. The value of this function diminishes as the state of the system is farther away from the sliding hyperplane. This allows to lessen the greatest value of the controller action, without increasing the chattering in the system. Moreover, the obtained reaching law was further altered, by removing the discontinuous terms. In this way, the switching quasi sliding motion is no longer ensured. However, as was demonstrated in the work, this also scales down the size of the quasi sliding mode band, and therefore enhances the robustness of the system. In the paper [9], the non-switching reaching law postulated in [8] was utilized to control data flow in a communication network. It has been shown, that this approach ensures some important practical properties, such as preventing the overflow in data buffers. On the other hand, in [10] the same reaching law was applied to the problem of generating resupply orders in a logistic system.

In the paper [21] a sliding mode controller for oscillation damping of fluid in a tank was considered. The fluid motion was modelled by a pendulum. The following exponential reaching law was proposed

$$s\left[(k+1)T\right] = \left(\beta_1 \exp\left(-1/\left|s(kT)\right|\right) + \beta_2\right) s(kT), \qquad (7)$$

where β_1, β_2 are positive parameters, the sum of which does not exceed 1 and $\beta_2 < \beta_1$. The reaching law (7) was compared to the ones presented in [3] and [37] using computer simulations. It has been shown, that (7) offers the smallest values of the Euclidean norm and maximum value of the controller effort. In [20] the reaching law (7) is used to design two sliding mode controllers for a two axis (one controller per axis) gimbal infrared scanner positioned on a missile. The scanner is made to follow a spiral trajectory to find the target of the missile in the final part of its flight. The designed controllers are then augmented by a disturbance estimator to improve the system robustness. The feasibility of the approach is tested in computer simulations.

In [23] the authors propose the following reaching law

$$\begin{aligned} s\left[(k+1)T\right] &= (1-qT)\Phi(kT)s(kT) - \lambda \operatorname{sgn}\left[s(kT)\right]/\Phi(k) + \\ &+ c^T\left\{f(kT) - 2f\left[(k-1)T\right] + 2f\left[(k-2)T\right]\right\}. \end{aligned} \qquad (8)$$

in which $q \in (0, 1/T)$, $\lambda > 0$. The nonlinear term $\Phi(k) \in (\delta,1)$ for some $\delta > 0$, tends to one, as $s(k)$ gets smaller, in order to achieve a better compromise between fast reaching phase and small chattering. The expression in the second line of (8) is used for better compensation of disturbance f ($\dim f = n \times 1$), under the assumption, that the second difference of f is bounded. This is an interesting approach, as in most works only bounds on the first difference of disturbance are considered. It is worth to point out, that in spite of using a reaching law that is somewhat similar to (1), the authors of [23] do not require the state of the plant to cross the switching hypersurface in each step. The authors, using computer simulations and laboratory tests of a piezoelectric actuator compare their approach to the one proposed in [33]. The results confirm a better robustness of the proposed algorithm.

Similarly as in [1] and [3], in the work [31] the authors omitted the requirement of crossing the sliding hypersurface in every consecutive step of the quasi sliding mode. The subsequent conditions for enforcing the quasi sliding mode were presented:

$$\begin{aligned} &s(kT) > \varepsilon \Rightarrow 0 < s\left[(k+1)T\right] < s(kT) \\ &s(kT) < -\varepsilon \Rightarrow s(kT) < s\left[(k+1)T\right] < 0 \\ &\left|s(kT)\right| \le \varepsilon \Rightarrow \left|s\left[(k+1)T\right]\right| \le \varepsilon. \end{aligned} \qquad (9)$$

According to the authors, if (9) hold, finite time convergence to the region $|s(kT)| \le \varepsilon$ is guaranteed. However, after studying the above conditions in more detail, it is clear, that they alone do not guarantee the convergence to a neighborhood of the switching hypersurface at all. It can be the case, that $\lim_{s(kT)\to s_g^+} \left\{s\left[(k+1)T\right] - s(kT)\right\} = 0$ for some $s_g > \varepsilon$. Then, if the initial value of the switching variable is greater than s_g, the switching variable will converge asymptotically to s_g. Therefore, the representative point will not enter the vicinity of the sliding hyperplane at all. An example of a reaching law, that satisfies conditions (9) is

$$s\left[(k+1)T\right] = \begin{cases} s(kT) - q\left[s(kT) - s_g \operatorname{sgn}\left[s(kT)\right]\right] & \text{for } \left|s(kT)\right| > s_g \\ (1-q)s(kT) & \text{for } \left|s(kT)\right| \le s_g \end{cases} \qquad (10)$$

One can observe, that for any given value of the sliding variable, the absolute value of this variable in the next time instant is strictly lower. However, if the initial value of the sliding variable is outside of the band $(-s_g, s_g)$, it will never enter this band, but will only monotonically converge to $-s_g$, or s_g. The subsequent reaching law is proposed:

$$s\left[(k+1)T\right] - s(kT) = d(x,kT) - d_0 - d_s \operatorname{sgn}\left[s(kT)\right], \qquad (11)$$

where $d(x, kT)$ is the (not known) impact of the disturbance, d_0 is the mean value of this term and d_s is the greatest possible discrepancy between $d(x, kT)$ and its mean. Thus, (11) can be considered as a special case of (1), in which $q = \varepsilon = 0$ and the convergence is ensured only by overestimating d_s. The action of the sliding mode controller designed according to (11) is demonstrated in computer simulations.

In the work [11] a general form of a reaching law is postulated. It guarantees the convergence of the representative point (state) to a known region around $s(kT) = 0$ and remaining in this band. Unfortunately, the authors do not present the application of this rule to any control system, they only consider an abstract evolution of the sliding variable.

A comparison of many above-mentioned control algorithms [1], [3], [14], [16], [17], [27] using servomechanism control as an example application was performed in the work [28].

IV. Conclusion and Future Work

In this work, a number of latest results on reaching law based sliding mode control for discrete time plants were discussed and compared. This approach has some important advantages, such as better control of system dynamics and state constraints during the reaching phase. Although this method was first proposed quite some time ago, many novel reaching laws, that are suited for the needs of particular control systems, still appear in the literature.

One of the promising directions of future research seems to be the reaching laws for sliding variables with increased relative degrees. This methodology is able to reduce the size of the quasi sliding mode (thus enhancing the robustness of the plant), without requiring increased control magnitude. It can also prove very useful if measurements of states directly influenced by the control signal are not available.

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